

Reduction of conducted EMI DC-DC Full Bridge Voltage Converter by a Dual Randomized PWM Scheme

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Abstract: Randomized Pulse Width Modulation (RPWM) deals better than Deterministic PWM (DPWM) with Electro-Magnetic Compatibility (EMC) standards for conducted Electro-Magnetic Interferences (EMI). In this paper, we propose a dual RPWM scheme for full bridge DC-DC voltage converter. This scheme is based on a triangular carrier with two randomized parameters. By using directly the randomized parameters of the carrier, a mathematical model of the Power Spectral Density (PSD) of output voltage is developed. Then, the PSD analysis shows the EMC advantage of the proposed dual randomization scheme comparatively to the classical simple randomization schemes. Finally, FFT analysis confirms this advantage.

Keywords-electromagnetic compatibility, DC-DC converter, RPWM, power spectral density

Introduction

Deterministic Pulse Width Modulation (DPWM) generates discrete harmonics with important magnitudes. The Electro-Magnetic Compatibility (EMC) standards impose more and more filtering effort in power converters [1]. One of the solutions for relaxing the filtering effort is the RPWM technique which deals better with EMC standards by spreading the voltage spectrum in a large frequency range and reducing its magnitude [2-4]. Several works regarding this new technique has been published lately, principally two randomization schemes are proposed; RCFM (Randomized Carrier Frequency Modulation) and RPPM (Randomized Pulse Position Modulation), for DC-DC conversion [4, 5] and for DC-AC conversion [2, 3, 5, 6]. Combinations of two randomized parameters have been also applied to the buck converter [1, 7] and to the three phase full bridge inverter [8].

In order to obtain a more spread spectrum, in full bridge DC-DC voltage converter, we propose a combination of two simple RPWM schemes (RCFM and RPPM schemes) that we call RCFM-RPPM or dual RPWM scheme. This scheme is based on a triangular carrier with two randomized parameters. First, we propose the modulating principle and then we develop an analytical model of PSD of output voltage. This model is expressed directly upon the random parameters

of the carrier and the simple RPWM schemes (RCFM scheme and RPPM scheme) can be found as particular cases of the general model (RCFM-RPPM). The PSD analysis shows that the proposed dual RPWM scheme allows a better spread PSD comparatively to the simple RPWM schemes that is the desired EMC advantage. Finally, the FFT analysis of output voltage confirms this advantage.

Modulating principle

The full bridge converter is schematized in Fig.1; it requires two switching functions q_a and q_b .

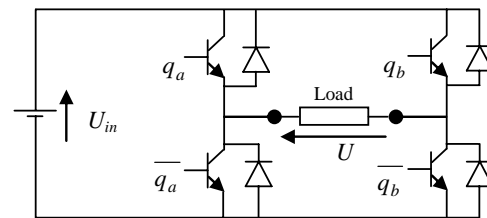


Figure 1. Full bridge DC-DC voltage converter

The modulating principle is illustrated in Fig.2; the two switching functions q_a and q_b are generated by comparison of two deterministic reference signals r_a and r_b of magnitudes d_a and d_b respectively, to a single randomized triangular carrier c . Generally, the magnitudes d_a and d_b are taken as follows:

$$\begin{cases} 0 < d_a < 1 \\ 0 < d_b < 1 \\ d_a + d_b = 1 \end{cases} \quad (1)$$

At low frequencies, the switching effects of power components are generally neglected [9-11], thus the output voltage U is expressed related to the input voltage U_{in} and the switching functions q_a and q_b as follows:

$$U = (q_a - q_b)U_{in} \quad (2)$$

Each of the two switching functions q_a and q_b is characterized by three parameters respectively (Fig.2): the switching period T , the duty cycle (d_a or d_b) and the delay report (δ_a or δ_b). In RPWM, these three

parameters should be randomized in a combined or a separated way. In practice, d is generally deduced from a deterministic reference signal giving the control of the output voltage U . Thus, only the switching period T (i.e. the period of the carrier) and the delay reports (δ_a and δ_b) can be really randomized.

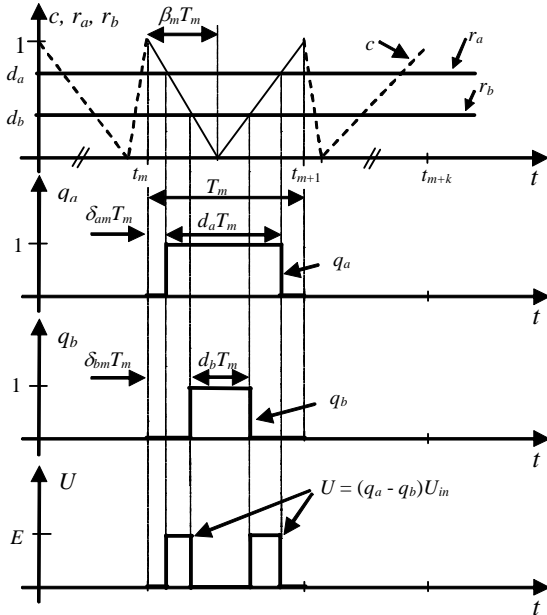


Figure 2. Modulating principle

From (Fig.2.), δ_a and δ_b are obtained as follows:

$$\delta_a = \beta(1 - d_a) \quad (3)$$

$$\delta_b = \beta(1 - d_b) \quad (4)$$

β : fall time report of the carrier c , (Fig.2).

From equations (3) and (4), a randomization of β in the interval $[0, 1]$ gives random delay reports (δ_a and δ_b) in the intervals $[0, (1 - d_a)]$ and $[0, (1 - d_b)]$ respectively and the resulting positions of the switching functions vary randomly from the beginning to the end of the period, (Fig.2). Thus, the RPPM scheme uses a carrier with fixed period T and randomized fall time report β (Fig.2). The particular case of Random Lead Lag Modulation (RLLM) requires two discrete random values of β with equal probability p_β : ($\beta = 0$ or $\beta = 1$, $p_\beta = 0.5$). We notice that the principal advantage of this particular scheme is the reduction of the switching losses [5].

For RCFM, the randomization limits T_{min} and T_{max} of T are generally fixed around a mean value \bar{T} : this scheme needs a carrier with randomized period T and fixed fall time report ($\beta = 0.5$).

The proposed dual RPWM scheme (RCFM-RPPM) combines the two previous schemes; carrier parameters (T and β) are independently randomized in the intervals defined for the two simple RPWM schemes respectively (RCFM and RPPM).

The resulting schemes are summarized in Table 1.

Table 1. Resulting RPWM schemes (α, β fixed: $\beta = 0.5$)

PWM Scheme	β	T
DPWM	fixed ^a	fixed
RPPM	randomized	fixed
RCFM	fixed ^a	randomized
RCFM-RPPM	randomized	randomized

Mathematical model of PSD

Spectral analysis of RPWM voltage can be performed either by FFT or by PSD:

- For the FFT computation, a τ length sample of the voltage is required (the signal is assumed to be periodic with period τ). From a statistical point of view, the result is not exact; it depends on the time-length τ . However, several studies using the FFT of random signals are performed in RPWM [2, 3].
- The PSD is a statistical parameter of random signals (i.e. Fourier transform of the autocorrelation); it is particularly useful in information theory because it leads to exact statistical characteristics [12, 13]. However, to set a mathematical model of PSD isn't a simple task for all RPWM schemes and for complex structures of the converters.

The PSD $W(f)$ of a random signal $u(t)$ is [13]:

$$W(f) = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} E \left[|F(u_\tau(t))|^2 \right] \quad (5)$$

Where:

$u_\tau(t)$: Considered signal during the time interval τ .

$F(u_\tau(t))$: Fourier transform of $u_\tau(t)$.

$E[\cdot]$: Statistical expectation.

For a random pulse signal, belonging to the class of Wide Sense Stationary (WSS) signals, expression (5) leads to the expression (6), [4-8]:

$$W(f) = \lim_{N \rightarrow \infty} \frac{1}{T} E \left[\sum_{k=-N}^N U_m(f) U_{m+k}^*(f) \right] \quad (6)$$

\bar{T} : Statistical mean of the switching period.

$U_m(f)$: Fourier transform of the pulse signal during an arbitrary switching period T_m .

$U_{m+k}^*(f)$: Complex conjugate of $U_{m+k}(f)$.

The expression (6) is developed as follows [4-8]:

$$W(f) = \frac{1}{T} \left\{ E \left[|U_m(f)|^2 \right] + 2 \operatorname{Real} \left(\sum_{k=1}^{\infty} E \left[U_m(f) U_{m+k}^*(f) \right] \right) \right\} \quad (7)$$

$\operatorname{Real}(\cdot)$: Real-part of the expression in brackets.

In the per unit system, U_{in} is equal to unity and the expression (2) of output voltage U becomes:

$$U = q_a - q_b \quad (8)$$

Thus, during the switching period T_m , Fourier transform $U_m(f)$ of the per unit output voltage U is:

$$U_m(f) = q_{a,m}(f) - q_{b,m}(f) \quad (9)$$

Where, $q_{a,m}(f)$ and $q_{b,m}(f)$ are given by (Fig.2):

$$q_{a,m}(f) = \frac{1}{\pi f} e^{-j\pi f d_a T_m} \sin(\pi f d_a T_m) e^{-j2\pi f \beta_m (1-d_a) T_m} e^{-j2\pi f t_m} \quad (10)$$

$$q_{b,m}(f) = \frac{1}{\pi f} e^{-j\pi f d_b T_m} \sin(\pi f d_b T_m) e^{-j2\pi f \beta_m (1-d_b) T_m} e^{-j2\pi f t_m} \quad (11)$$

Replacing $U_m(f)$ and $U_{m+k}^*(f)$ by their expressions in (7) and considering the lag time between the m^{th} and the $m^{\text{th}+k}$ switching periods (Fig. 2) as follows:

$$t_{m+k} - t_m = T_m + (t_{m+k} - t_{m+1}) = T_m + \sum_{l=m+1}^{m+k-1} T_l = T_m + \gamma_k \quad (12)$$

We obtain the following expression of the PSD:

$$W(f) = \frac{1}{T} \left\{ E_T \left[|U(f)|^2 \right] + 2 \operatorname{Real} \left(\frac{E_{T,\beta} \left[U(f) e^{j2\pi f T} \right] E_{T,\beta} \left[U^*(f) \right]}{1 - E_T \left[e^{j2\pi f T} \right]} \right) \right\} \quad (13)$$

Where:

$$U(f) = \frac{1}{\pi f} \begin{pmatrix} e^{-j2\pi f \beta (1-d_a) T} e^{-j\pi f d_a T} \sin(\pi f d_a T) \\ -e^{-j2\pi f \beta (1-d_b) T} e^{-j\pi f d_b T} \sin(\pi f d_b T) \end{pmatrix} \quad (14)$$

And, [13]:

$$E_T \left[e^{j2\pi f \gamma_k} \right] = \left(E_T \left[e^{j2\pi f T} \right] \right)^{k-1} \quad (15)$$

Particular case of RCFM scheme: The carrier has a fixed fall-time report ($\beta = 0.5$) and a randomized period T , which gives:

$$W(f) = \frac{1}{T} \left\{ E_T \left[|U(f)|^2 \right] + 2 \operatorname{Real} \left(\frac{E_T \left[U(f) e^{j2\pi f T} \right] E_T \left[U^*(f) \right]}{1 - E_T \left[e^{j2\pi f T} \right]} \right) \right\} \quad (16)$$

Particular case of RPPM scheme: The carrier has a fixed period T and a randomized fall-time report β , the resulting PSD expression is:

$$W(f) = \frac{1}{T} \left\{ E_\beta \left[|U(f)|^2 \right] + 2 \operatorname{Real} \left(\frac{E_\beta \left[U(f) \right] E_\beta \left[U^*(f) \right] e^{j2\pi f T}}{1 - e^{j2\pi f T}} \right) \right\} \quad (17)$$

At $\left(f_k = \frac{k}{T}, k = 0, 1, \dots \right)$, the denominator of

expression (17) becomes ($1 - e^{j2k\pi} = 0$), and the PSD (in volt²/Hertz), has discrete components with infinite magnitudes. Thus, it is well suitable to decompose the expression (6) two terms: a continuous term (continuous PSD) and a discrete one (power harmonics), [4, 5]:

$$W(f) = \frac{1}{T} \left\{ E_\beta \left[|U(f)|^2 \right] - |E_\beta \left[U(f) \right]|^2 + \frac{1}{T} |E_\beta \left[U(f) \right]|^2 \sum_{k=-\infty}^{+\infty} \delta \left(f - \frac{k}{T} \right) \right\} \quad (18)$$

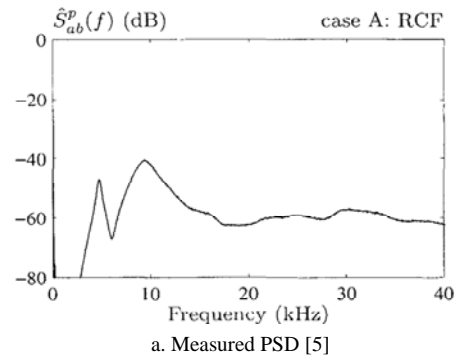
In case of DPWM, the PSD contains only the discrete part (power harmonics in “volt²”):

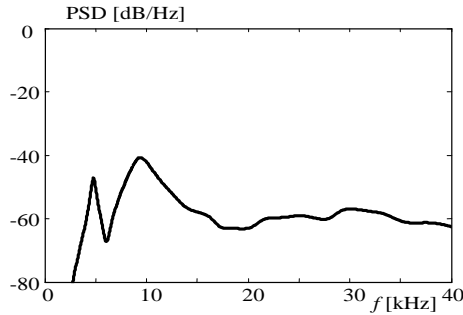
$$W(f) = \frac{1}{T^2} |U(f)|^2 \left(\sum_{k=-\infty}^{+\infty} \delta \left(f - \frac{k}{T} \right) \right) \quad (19)$$

$\delta(\cdot)$: Dirac pulse.

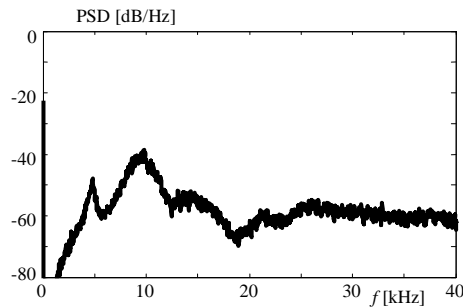
Validation of PSD model

RPPM and RCFM schemes: Computed PSDs are compared to the measure published in the literature and to those simulated by using WELCH approximation [14] in the same conditions. The results of Fig.3 reveal a perfect agreement between the computed PSDs by using the proposed models (expression 16 and expression 18), the measure published by *M. M. Bech* [5] and the simulation.



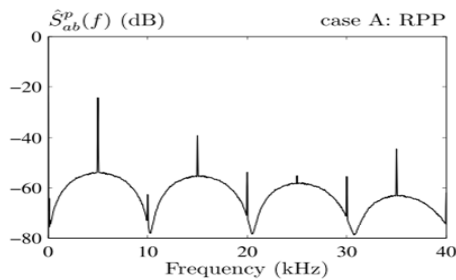


b. Computed PSD (expression 16)

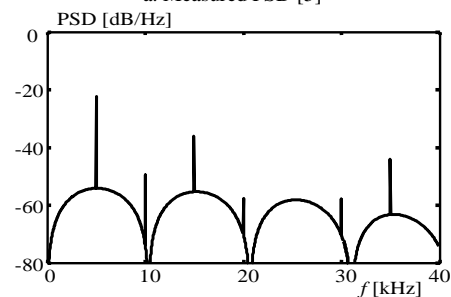


c. Simulated PSD (WELCH approximation)

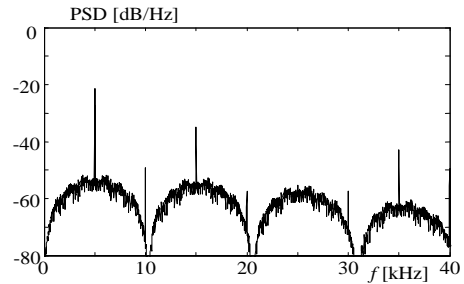
Figure 3. Validation of RCFM scheme



a. Measured PSD [5]



b. Computed PSD



c. Simulated PSD (WELCH approximation)

Figure 4. Validation of RPPM scheme (particular case of RLLM)

As suggested in the reference [5], d_a and d_b need to be corrected in order to compensate the blanking time between two complementary power switches of the same leg. However, a slight difference is still noted between the computation and the measure for RPPM scheme because of the DC link voltage source and the voltage drop across the power devices [5].

From Fig.4, RPPM scheme is not able to spread the PSD; it contains a continuous part (noise) and a discrete one (power harmonics), RCFM scheme gives a completely spread PSD that reduces considerably the magnitude (Fig.3), thus RCFM scheme gives more EMC advantages than RPPM scheme.

RCFM-RPPM scheme: This scheme is validated by comparing the analytical computation to the simulation (Welch approximation), in fig.5, in the same conditions of the previous schemes and by comparison to RPPM while decreasing progressively the randomization effect of the period T (Fig.6.a) and to RCFM while decreasing progressively the randomization effect of parameter β (Fig.6.b), under the following conditions:

- Input voltage is: $U_{in} = 1$ pu.
- RPPM scheme: T is fixed and β is randomized in the interval: $\left[\overline{\beta} \left(1 - \frac{R_\beta}{2} \right), \overline{\beta} \left(1 + \frac{R_\beta}{2} \right) \right]$, with $(\overline{\beta} = 0.5 \text{ and } R_\beta \leq 2)$.
- RCFM scheme: the parameter β is fixed, ($\beta = 0.5$) and the period T is randomized in the interval $\left[\overline{T} \left(1 - \frac{R_T}{2} \right), \overline{T} \left(1 + \frac{R_T}{2} \right) \right]$, R_T is the randomness level: theoretically, $(0 \leq R_T \leq 2)$.
- RCFM-RPPM: combination of two schemes RPPM and RCFM.
- All randomizations are performed by using the uniform probability distribution function.

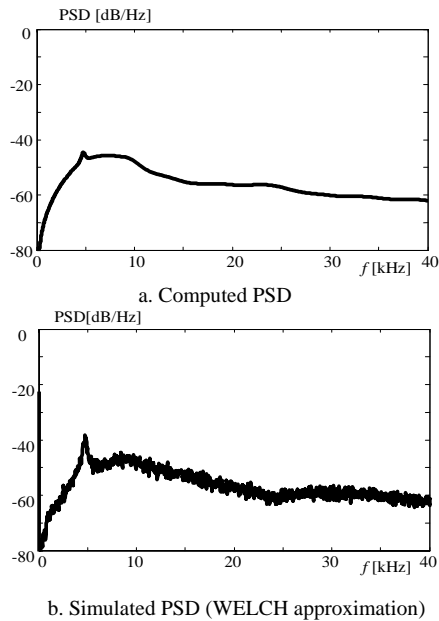


Figure 5. Comparison between the computation and the simulation

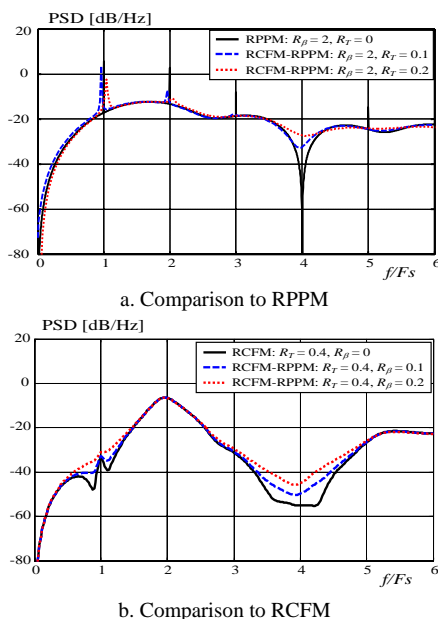


Figure 6. Comparison to simple randomization schemes

From Fig.6, RCFM-RPPM converges perfectly to RPPM while R_T decreases and to RCFM while R_β decreases, which reinforces the validity of the proposed PSD models for all schemes.

PSD analysis of the output voltage

Fig.7 shows the PSDs of output voltage with three values of duty cycle d ($d = 0.3$, $d = 0.5$ and $d = 0.8$). In order to show the EMC advantage of the proposed dual RPWM scheme, different values of R_β are considered

and the particular case of RCFM scheme ($R_T = 0.2$, $R_\beta = 0$) is taken as a benchmark.

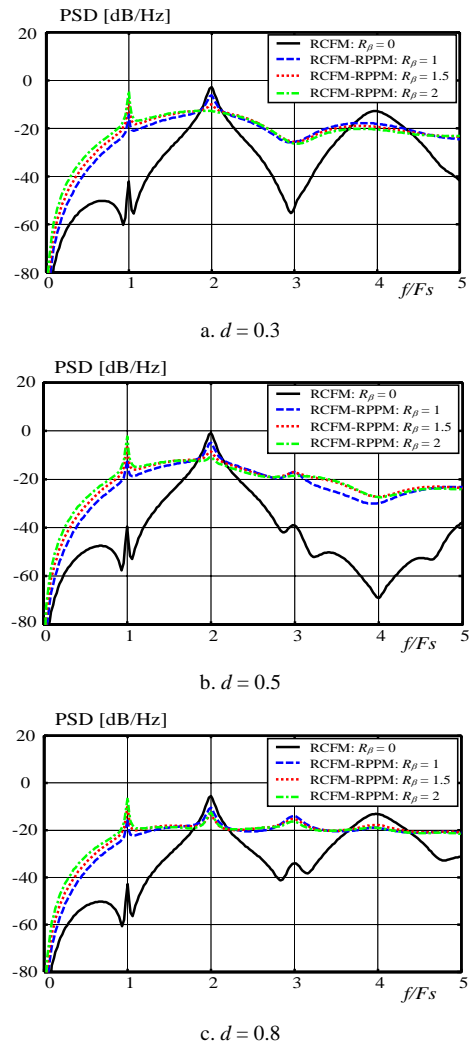


Figure 7. Voltage PSD

Globally, the randomization of β (Fig.7) adds an important spread to the PSD and a reduction of the picks at the multiples of the mean switching frequency (F_s).

The full bridge requires two switching functions q_a and q_b with duty cycles d_a and d_b satisfying the condition (1), thus the randomization of β gives two random delay reports (δ_a and δ_b) which adds an important spread to the PSD for all values of the duty cycles d_a and d_b (Fig.7). However, we notice that for important values of R_β ($R_\beta > 1.5$), the magnitude of the PSD pick at ($f = F_s$) becomes important. In order to obtain a compromise between the two picks' magnitudes at the frequencies ($f = F_s$ and $f = 2F_s$), we propose a reduction of the randomness level R_β :

($1 \leq R_\beta \leq 1.5$), (Fig. 7). Globally, the proposed RCFM-RPPM scheme adds an important spread to the PSD (Fig. 7) that is the desired EMC advantage.

FFT analysis of the output voltage

By use of MatLab-Simulink Software, the converter is simulated and the output voltage analyzed (Fig.8) under the following conditions:

- Input voltage: $U_{in} = 150$ v
- Load: R - L load ($R = 10 \Omega$, $L = 10$ mH)
- Reference signals: $d_a = 0.75$ and $d_b = 0.25$
- Parameters of the carrier:
 1. DPWM: T and β fixed, ($T = \frac{1}{F_s}$, $F_s = 1800$ Hz and $\beta = 0.5$).
 2. RPPM: T fixed ($T = \frac{1}{F_s}$, $F_s = 1800$ Hz) and β randomized ($\bar{\beta} = 0.5$, $R_\beta = 1.8$)
 3. RCFM: T randomized ($\bar{T} = \frac{1}{F_s}$, $F_s = 1800$ Hz and $R_T = 0.2$) and β fixed ($\beta = 0.5$).
 4. RCFM-RPPM: T and β randomized ($\bar{T} = \frac{1}{F_s}$, $F_s = 1800$ Hz and $R_T = 0.2$) and ($\bar{\beta} = 0.5$, two values of R_β are used: $R_\beta = 1.8$ and $R_\beta = 1.2$).

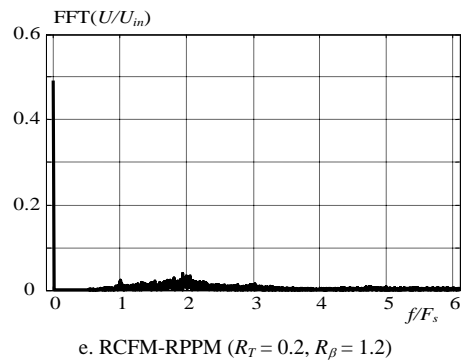
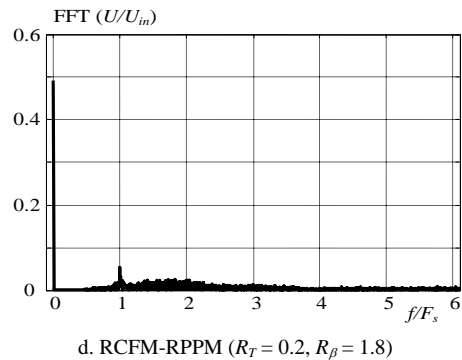
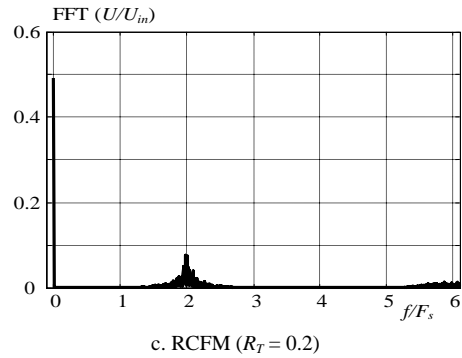
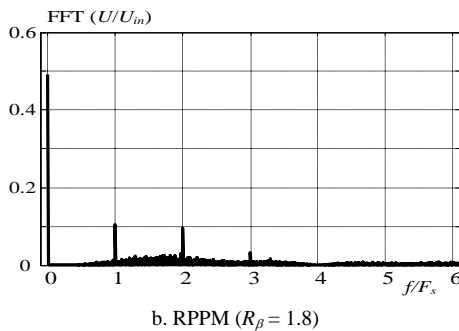
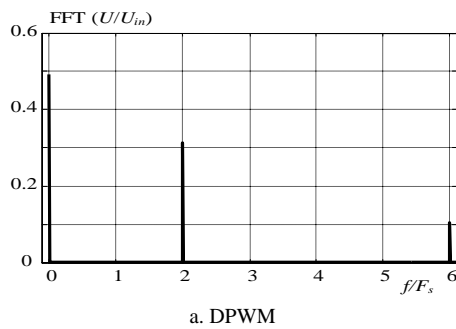


Figure 8. Voltage spectrums for different schemes

The results of Fig.8 are in conformity with the obtained PSDs for all schemes:

- DPWM: discrete spectrum.
- RPPM: the spectrum contains a continuous part (harmonics) and a discrete part (noise).
- RCFM: the spectrum is completely spread into a continuous noise with a relative important magnitude around the switching frequency F_s .
- RCFM-RPPM: more spread spectrum, however for important values of R_β , ($R_\beta = 1.8$), the magnitude around ($f = F_s$) becomes important. As proposed for the PSD, for $R_\beta = 1.2$, we obtain a compromise between the magnitudes of the

spectrum at ($f = F_s$) and at ($f = 2F_s$), (Fig.8.d and Fig.8.e).

Conclusion

In this paper, a dual RPWM scheme based on a triangular carrier with two randomized parameters is proposed for the full bridge DC-DC voltage converter. By using the randomized parameters of the carrier, a mathematical model of the PSD of output voltage is developed and validated. The PSD analysis shows clearly the EMC advantage of the proposed dual RPWM scheme comparatively to the simple RPWM schemes. Finally the FFT analysis confirms this advantage.

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