

## Investigation of Electromagnetic Field Interference of Power Electronic Converters

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**Abstract:** The use of power electronic converters is strongly increased in last years which increase the electromagnetic interference (EMI). In this work we propose to compute the electromagnetic field distribution for characterizing the electromagnetic perturbations emitted by the power converters circuits.

For this objective we use two different modeling; the first is based on the resolution of Maxwell equations by FDTD and the second by using the Hertzian dipole concept and the modified images theory. These two approaches are compared in term of the precision, implementation and computing time consuming. Also, we propose a comparison with FEKO software which is based on the antenna theory.

### Introduction

The fast and diversified diffusion of the power electronic equipments so much in the professional and military domain (embarked equipment, variation speed, heating, control of energy...) and for general public (domestic, automobile, computers...) multiply the electromagnetic sources of disturbance. These disturbances, sometimes very constraining, led the engineers of research towards a vast field commonly called (Electro-Magnetic Compatibility).

In this work, we are interested to the quantification by calculation of the electromagnetic field radiated by the power converters. Let us consider the complex shape of the power electronics converters, an accurate modeling requires taking into account the different components forming the converters.

Classically, numerical modeling in frequency domain is used with the marked code of simulation (NEC, FEKO ...); this modeling consists in the resolution of the integral equation by moment method [1], the Fast Fourier Transform permits the study in time domain.

In the literature, for working directly in time domain, the modeling consists on the resolution of Maxwell equations by FDTD [2].

In reality, the power electronic converters are subject to the transient current and voltages during regular functioning of the switch (turn-off and turn-on). For this reason, it's important to quantify the electromagnetic field emitted by the converters in time

domain and to know the signature of the perturbation source with its spectral content.

In our work we propose another approach for modeling this problem in time domain. This approach is based on the use of Hertzian dipole concept and modified images theory for taking into account the ground plan and the dielectric board. To calculate analytically the electromagnetic field by the dipole concept it's require to know the current distribution. For this objective, using FDTD, we deduce and solve a matrix equation.

Our proposition allows taking into account the non linearity introduced by real functioning of the switches (controlled transistor). Finally, we propose a comparison between the two methods; this comparison takes into account the precision, computing time, and difficulty of implementation...

### Resolution of Maxwell equations by FDTD

The geometry of a typical power converter circuit (buck converter) is illustrated in figure 1, where  $V_s$  is the voltage source,  $R_s$  is source resistor,  $S_w$  denotes the switch component, and  $R$  and  $L$  represent the resistor and inductance of the load respectively and while  $D$  represents a free-wheel diode, the space discretization is taken 2 mm in 3D where the unknown parameter are  $E$  and  $H$  given in Maxwell curl equations (1) and (2).

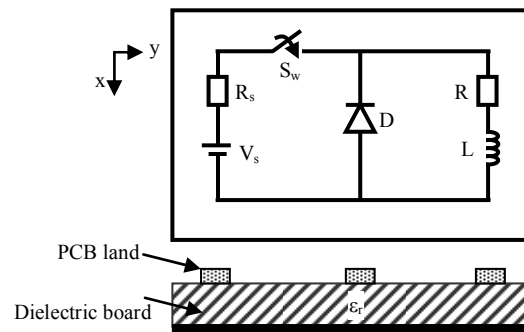


Figure 1: The geometry of power converter.

Considering the geometry of power electronic converter (figure 1), the computation of electromagnetic field distribution radiated by this last is not a fast task, this is due to the fact that it contains non linear lumped

elements (Diode, transistor), dielectric layer and PCB traces. In time domain, numerical computation can be obtained based on the resolution of Maxwell equations by FDTD method. The Maxwell curl equations can be written as:

$$\text{curl}\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\text{curl}\vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (2)$$

K. S. Yee [2] introduces a set of finite difference equations for the system of scalar equations equivalent to Maxwell's equations in the Cartesian coordinate system, and to realize all of the space derivatives. He evaluates E and H at alternate half-time steps.

Due to the finite capabilities of the computers used to implement the FDTD method, the mesh must be limited in the x, y, and z directions. The Maxwell curl equations discretized using central-difference cannot be used to evaluate the electric field components tangential to the outer boundaries since they would require the values of field components outside of the mesh [3].

**Absorbing boundary conditions:** In the configurations treated in this paper, one of the six mesh boundaries of computation domain is a perfectly ground plane and its tangential electric field values are forced to be zero [4]. The tangential electric field components on the other five mesh walls must be specified in such a way that outgoing waves are not reflected using the absorbing boundary condition [3].

**Handling components for applying FDTD method:** FDTD method is extensively used for calculating electromagnetic problems since K. S. Yee first proposed it in 1966 [2]. We apply FDTD method in 3-D for calculating the electromagnetic fields distribution. In this case, some problems must be solved. First, dealing with the printed circuit traces; second, handling the dielectric boards; in third, some kinds of lumped loads, including linear loads, like resistor, capacitor and inductance, and nonlinear loads, such as diode and electronic switch. This approach is largely described in literature [5].

**Dealing with the dielectric board:** In 1988, A. Taflove [6] first proposed a method, contour integral approach derived from the Maxwell's integral equations. This approach permits us to derive the FDTD equation in an inhomogeneous medium, which is, including the free space and dielectric board.

**The printed circuit traces :** The PCB trace has a very high conductivity, so these electric conductors can be assumed to be perfectly conducting and have zero thickness, and simply treated by setting the electric field components that lie on the conductors to zero [4]. Through numerical experiment testing, it is demonstrated that this method is suited to our applications.

**Linear or nonlinear lumped components:** The power converter circuit treated in this paper (Figure 1) includes linear lumped elements (resistor, inductance) and nonlinear lumped elements (diode and transistor). Using FDTD method, lumped elements may be accounted in Maxwell's equation by starting with Ampere's equation [5].

## Electromagnetic Field Computation by Analytical Approach

**Hertzian dipole method:** The Hertzian dipoles concept consists in a segmentation of antenna into electrically small elements named dipole (where the rayon must be very small then her length). The length of the dipole must satisfy the two following conditions [7]:

1.  $dz \leq \lambda/20$

This condition permits to mask the propagation along the dipole, which means that both of the amplitude and the phase of the current along the dipole are constants.

dz: the length of a dipole;

$\lambda$ : the wavelength in frequency domain.

2.  $dz \leq r/10$

This condition permits to take in account the small variations of the current seen very close to the dipole.

r: the point of the field computation.

The expression of the magnetic vector potential at a point M (x, y, z) is:

$$d\vec{A}_z = \frac{\mu_0}{4\pi r} I(z, t - \frac{r}{c}) dz \vec{u}_z \quad (3)$$

Also knowing that:

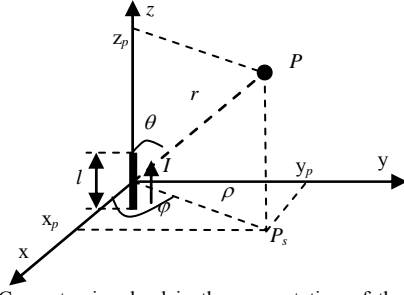
$$\vec{E} = -\vec{\text{grad}} V - \frac{\partial \vec{A}}{\partial t} \quad (4)$$

$$\vec{B} = \text{curl}\vec{A} \quad (5)$$

$$\Delta \vec{A} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{A}}{\partial z^2} = 0 \quad (6)$$

We can deduce  $\vec{E}$  and  $\vec{H}$  components [7].

Figure 2 shows the Hertzian dipole model and geometrical parameters.



**Figure 2:** Geometry involved in the computation of the fields with Hertzian dipoles.

This concept is done in free space, the ground plan and dielectric layer are tacked into account by using the modified images theory [9], and the total field is obtained by superimposing of each dipole contribution.

For calculating the transient electromagnetic field by Hertzian dipole concept, it's necessary to knowing the current distribution in the converter circuit. In the next step, we describe the adopted method for the current calculation.

**Current and voltage computation:** For calculating the currents and voltages distribution, we solve a matrix equation  $f([X]) = [0]$  that can be linear or no. The matrix equation can be expressed as follow:

$$f([X]) = [A][X] - [B] = [0] \quad (6)$$

The part  $[A][X]$  is linear, on the other hand  $[B]$  can be a nonlinear function  $g([X])$ .

In this section, we use a formalism based on the discretization of the transmission lines equations by the FDTD method [8]. This first step allows us to define the  $[A]$  matrix composed of two sub-matrices  $[A1]$  and  $[A2]$  as:

$[A]$ : matrix of topological representation of the circuit;  
 $[A1]$ : sub-matrix deduced from the representation of the propagation tubes (coupled transmission line);  
 $[A2]$ : sub-matrix deduced from the Kirchoff's laws (KCL and KVL) for the junctions (extremities and interconnections networks);

The transmission line equations are given by (7) and [8]:

$$\frac{\partial[V]}{\partial z} = [R][I] + [L]\frac{\partial[I]}{\partial t} \quad (7)$$

$$\frac{\partial[I]}{\partial z} = [G][V] + [C]\frac{\partial[V]}{\partial t} \quad (8)$$

While replacing the spatial and temporal derivatives by a finite difference, we deduce the recurrence equations for the voltage (9) and for the current (10):

$$[V_k^n] = \left[ \frac{[C]}{\Delta t} + \frac{[G]}{2} \right]^{-1} \left[ \left[ \frac{[C]}{\Delta t} - \frac{[G]}{2} \right] [V_k^{n-1}] - \frac{[I_k^{n-1/2}] - [I_{k-1}^{n-1/2}]}{\Delta z} \right] \quad (9)$$

where :  $k = 2, 3, \dots, k_{\max} - 1$

$$[I_k^{n+1/2}] = \left[ \frac{[L]}{\Delta t} + \frac{[R]}{2} \right]^{-1} \left[ \left[ \frac{[L]}{\Delta t} - \frac{[R]}{2} \right] [I_k^{n-1/2}] - \frac{[V_{k+1}^n] - [V_k^n]}{\Delta z} \right] \quad (10)$$

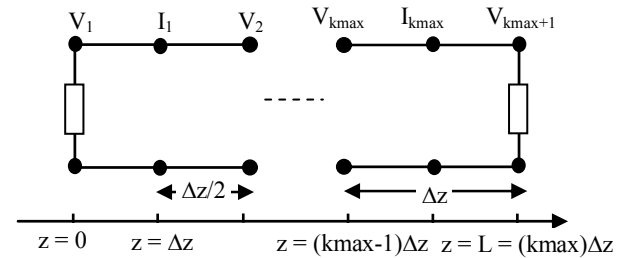
where :  $k = 1, 2, \dots, k_{\max} - 1$

Where:  $[R]$ ,  $[L]$ ,  $[C]$  and  $[G]$ : the matrixes of per unit length parameters.

For the calculation of the per unit length parameters of PCB lands, C. R. Paul [9] proposes a concept based on the resolution of Laplace equation by the moments method and the use of images method [9].

C. R. Paul starts with deducing the potential coefficients matrix, then, the inductance matrix is deduced by using an expression between capacitances and inductances matrixes [9]. This step allows us obtaining the capacitances and the inductances of the PCB with or without ground plane.

**1. Construction of matrix  $[A]$ :** While using the FDTD method, we suppose that the currents and the voltages don't coexist at the same point of the space, figure 3.



**Figure 3:** The spatial discretization of the line.

In order to obtain a matrix equation where the unknowns are the currents and the voltages at the two extremities (i.e.  $z = 0$  and  $z = L$ ) at the instant  $t = n.\Delta t$ , we proceed like as:

In the equation (9) we replace  $k$  by 1 and  $k_{\max}$  respectively while substituting  $\Delta z$  by  $\Delta z/2$ , and we introduce a temporal average for the current, we obtain:

- For  $k=1$

$$\begin{bmatrix} [C] \\ \Delta t \end{bmatrix} + \frac{[G]}{2} \left[ V^n(0) \right] - \frac{[I^n(0)]}{\Delta z} = \begin{bmatrix} [C] \\ \Delta t \end{bmatrix} - \frac{[G]}{2} \left[ V^{n-1}(0) \right] + \frac{[I^{n-1}(0)]}{\Delta z} - \frac{[I_1^{n-1}]}{\Delta z/2} \quad (11)$$

- For  $k = k_{max}$

$$\begin{bmatrix} [C] \\ \Delta t \end{bmatrix} + \frac{[G]}{2} \left[ V^n(L) \right] - \frac{[I^n(L)]}{\Delta z} = \begin{bmatrix} [C] \\ \Delta t \end{bmatrix} - \frac{[G]}{2} \left[ V^{n-1}(L) \right] + \frac{[I^{n-1}(L)]}{\Delta z} - \frac{[I_{k_{max}-1}^{n-1}]}{\Delta z/2} \quad (12)$$

The contribution of a tube with N traces is:

$$[A_1] = \begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots \\ \vdots & \begin{bmatrix} [C] \\ \Delta t \end{bmatrix} + \frac{[G]}{2} & - \frac{[I_N]}{\Delta z} & [0] & [0] & \vdots \\ \vdots & [0] & [0] & \begin{bmatrix} [C] \\ \Delta t \end{bmatrix} + \frac{[G]}{2} & \frac{[I_N]}{\Delta z} & \vdots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \quad (13)$$

Where:  $[I_N]$  is identity matrix of order N.

Based on this equation setting, we obtain a matrix of  $(2*N)$  equations with  $(4*N)$  unknowns. The matrix equation is completed by writing equations in the extremities and/or interconnections nodes which are regrouped in the sub-matrix  $[A_2]$ . The sub-matrix  $[A_2]$  is constructed while adopting the relation (14):

$$\sum_{j=1}^n \left( [Y_j^m] [V_j^m] + [Z_j^m] [I_j^m] \right) - [P^m(V_j^m, I_j^m)] = 0 \quad (14)$$

where:

$[P^m(V_j^m, I_j^m)]$ : vector of voltage and/or current sources.

$[Y_j^m]$ ,  $[Z_j^m]$ : resulting matrix from the use of Kirchhoff's laws at the node m, which contain the admittances or impedances respectively and values of 0, 1 or -1;

**2. Vector of the unknowns [X]:** The vector of the unknowns (13) contains the voltages and currents at the two extremities of the tubes constituting the converter. The contribution of the  $i^{th}$  tube at the instant  $t = n.\Delta t$  is:

$$[X] = \left[ \dots \quad [V_i^n(0)] \quad [I_i^n(0)] \quad [V_i^n(L)] \quad [I_i^n(L)] \quad \dots \right]^T \quad (15)$$

**3. Vector [B]:** The vector [B] is composed by two sub-vectors (16):

$$[B] = \begin{bmatrix} [B_1] \\ [B_2] \end{bmatrix} \quad (16)$$

**a. The sub-vector [B<sub>1</sub>]:** Each second member of equations (11) and (12) contains the terms calculated at the previous instant and permits the construction of the sub-vector  $[B_1]$ .

The contribution of a tube of the converter is the following:

$$[B_1] = \begin{bmatrix} \vdots \\ \begin{bmatrix} [C] \\ \Delta t \end{bmatrix} - \frac{[G]}{2} \left[ V^{n-1}(0) \right] + \frac{[I^{n-1}(0)]}{\Delta z} - \frac{[I_1^{n-1}]}{\Delta z/2} \\ \begin{bmatrix} [C] \\ \Delta t \end{bmatrix} - \frac{[G]}{2} \left[ V^{n-1}(L) \right] + \frac{[I^{n-1}(L)]}{\Delta z} - \frac{[I_{k_{max}-1}^{n-1}]}{\Delta z/2} \\ \vdots \end{bmatrix} \quad (17)$$

**b. The sub-vector [B<sub>2</sub>]:** For the sub-vector  $[B_2]$ , in addition to the current and/or voltage sources, the Kirchhoff's laws, written for nonlinear elements make appear supplementary terms. These last are introduced in the sub-vector  $[B_2]$ , which allows us to obtain a nonlinear system that we will illustrate in the numerical simulation section.

**4. Resolution of the matrix equation :** The resolution of the nonlinear matrix equation at every time step  $\Delta t$  gives the currents and voltages in every node of the network, the recurrence equations (11) and (12) permit us to deduce these last along the conductor (discretization point) of every tube. Next, we calculate the electromagnetic field using the Hertzian dipole concept.

## Numerical Simulation

The analyzed circuit is illustrated in figure 4 both for the cases with and without ground plane.

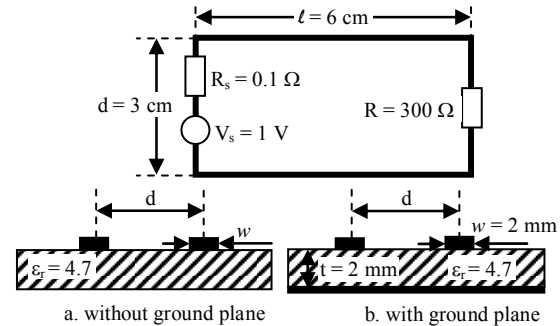


Figure 4: Configuration of the circuits used for simulation.

At first, we treat an ideal switch, and the turn-on process is modeled by taking a step signal ( $V_s$ ) with rise time  $t_r = 0.5$  ns.

**Circuit without ground plane:** In figures 5 and 6 we present the result of electric and magnetic field variation in the center of the circuit and at 5 cm above the circuit board.

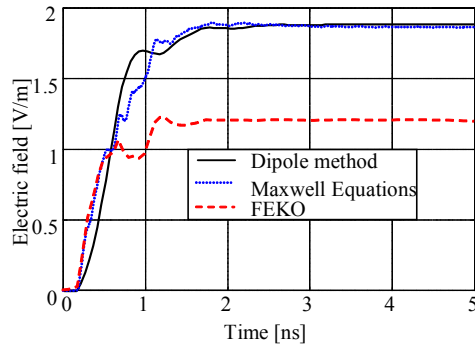


Figure 5: Electric field radiated by the circuit without ground plane.

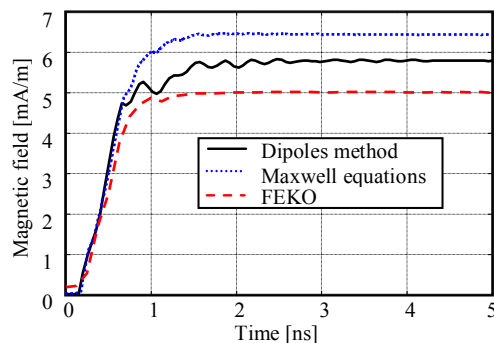


Figure 6: Magnetic field radiated by the circuit without ground plane.

The two approaches directly in time domain (Hertzian dipoles and the resolution of the Maxwell's equations by FDTD) give very close results in shape and amplitude (figures 5 and 6). The slight difference between the two calculations is certainly the result of spatial and temporal discretizations when using the FDTD and the superposition principle to the concept of dipoles.

Using FEKO software [10] and Fourier transform for modeling in time domain, we remark that the general shape is conserved but the magnitude is decreased at the end of rise time (established regime). This result is predictable because the use of the FFT for the transition frequency-time, and vice versa requires taking into account very special precautions (sampling frequency, sufficient number of points, continued decreasing...), which otherwise can seriously affect the results.

**Circuit with ground plane:** For this simulation, we consider the geometric configuration in figure 5 with ground plane (b) and the same point for calculating the electromagnetic field.

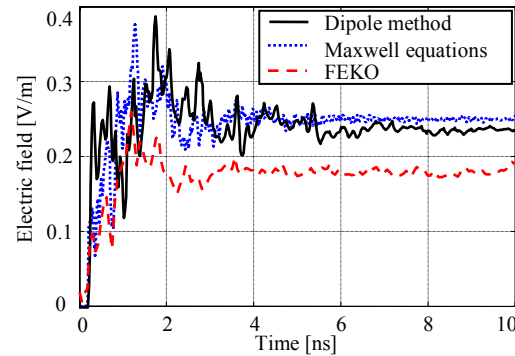


Figure 7: Electric field radiated by the circuit with ground plane.

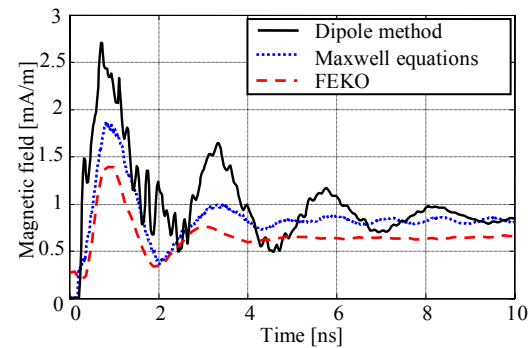


Figure 8: Magnetic field radiated by the circuit with ground plane.

The introduction of the ground plan highlights the significant difference between the three formalisms. The difference between the calculations made directly in time and this using software FEKO can be justified by the use of the Fourier transform. For the magnetic field we see a slight difference between the two methods temporal; this difference shows that the method of multiple images adapts especially for the calculation of the electric field in the event that we have a perfectly ground plane.

**Power converter with non-linear operation (Buck converter):** We are interested in this application to the study of the radiation of a real converter (Buck converter) with perfectly ground plane (Figure 9). The physical and geometric parameters of the converter are given in Figure 9.

The switch is a bipolar transistor controlled by a rectangular pulse signal. For the mathematical representation of its operation, we use the model of Ebers-Moll [5] resulting from superimpose of modes F (Forward) and R (Reverse).

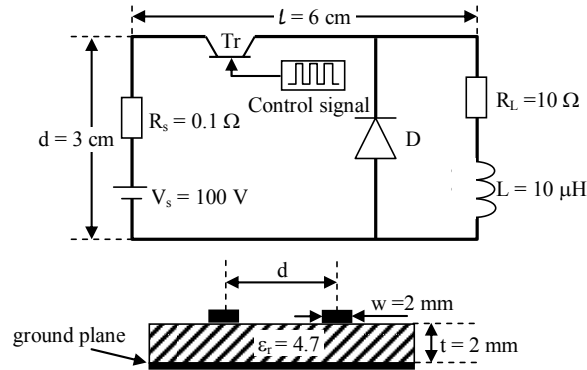


Figure 9: Electric scheme of the buck converter (with ground plane).

For describing the modeling of switches, we consider here the example in figure 10; we represent the electric scheme of the loop composed by the transistor centered between two traces (lines), the load and the source.

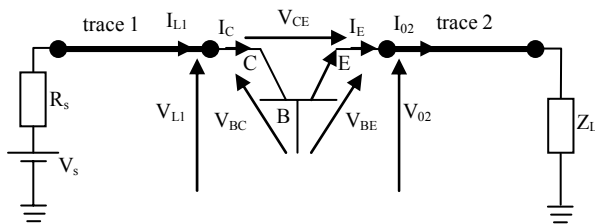


Figure 10: Electric scheme of simplified the converter.

The emitter and collector currents  $I_E$  and  $I_C$  are expressed by (18):

$$I_E = \alpha_R I_R - I_F \text{ and } I_C = I_R - \alpha_F I_F \quad (18)$$

Where:

$$I_F = I_0 \left( e^{(qV_{BE}/kT)} - 1 \right), \quad I_R = I_0 \left( e^{(qV_{BC}/kT)} - 1 \right) \quad (19)$$

k: Boltzmann constant;

T: temperature in degree Kelvin;

$I_0$ : saturation current;

$\alpha_R$  and  $\alpha_F$ : gains with modes R and F respectively.

For the computation of the currents distribution we propose to solve the matrix equation (6). In our case, (figure 10) the use of Kirchoff's laws at the node 1 and 2 give a nonlinear equation.

From figure 10, we have:

$$I_{L1} = I_C \text{ and } I_{O2} = I_E \quad (20)$$

From (18) and (19), while considering the transistor model (20) we obtain a nonlinear matrix equation (21):

$$f([X]) = [A][X] - g([X]) = [0] \quad (21)$$

Where:  $g([X])$  includes the unknown voltages ( $V_{L1}$ ,  $V_{O2}$ ) at the nodes 1 and 2 respectively (figure 10), which gives the non linearity of the matrix equation. This last are treated using by Newton-Raphson method.

$$[g([X])] = \begin{bmatrix} \vdots \\ I_0 \left( \alpha_F \left( \exp\left(\frac{V_{BE}}{V_T}\right) - 1 \right) - \left( \exp\left(\frac{V_{CE}}{V_T}\right) - 1 \right) \right) \\ I_0 \left( \left( \exp\left(\frac{V_{BE}}{V_T}\right) - 1 \right) - \alpha_R \left( \exp\left(\frac{V_{CE}}{V_T}\right) - 1 \right) \right) \\ \vdots \end{bmatrix} \quad (20)$$

The transistor parameters used in simulation are:  $\alpha_f = 0.99$ ,  $\alpha_r = 0.5$  and  $I_0 = 10^{-16} \text{ A}$ ,  $V_T = kT/q$ .

$V_{CE} = V_{BE} - V_{BC}$  and we denote the unknown  $V_{BC}$  and  $V_{BE}$  by  $V_{01}$  and  $V_{02}$  respectively, the figure 11 illustrates the voltage across the load  $V_{R-L}$ , and the control signal  $V_{BE}$  at the terminals of the BE junction transistor.

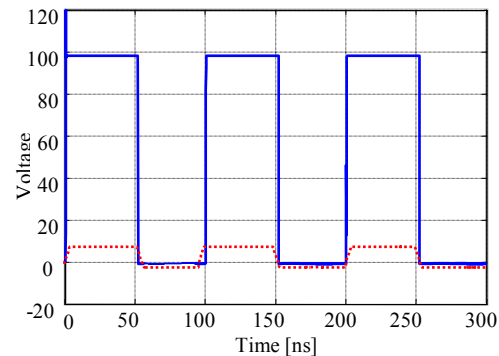


Figure 11: Load voltage (buck converter with ground plane).

For an observation point located at 5 cm above the circuit center, figures 12 and 13 show respectively the variations of electric and magnetic fields obtained using dipole concept in time domain.

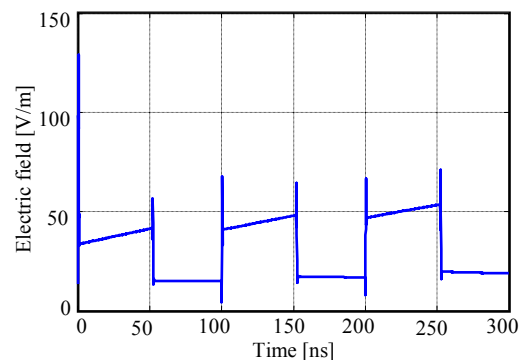
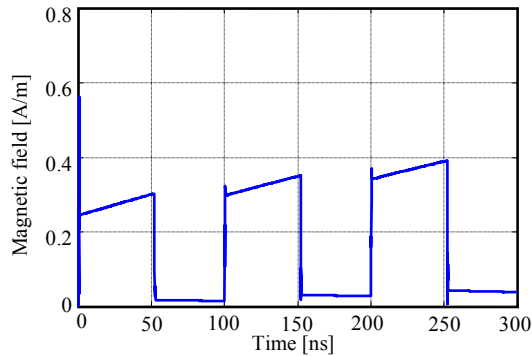


Figure 12: Electric field radiated by the buck converter.



**Figure 13:** Magnetic field radiated by the buck converter.

The results presented in figures 11 and 12 shows the presence of oscillations at commutations. We note also that the magnetic field is nonzero after the turn-off of the transistor, because the presence of the diode allows the current flow in the second loop and thus causes a magnetic field.

## Conclusion

In this paper we are interested to the calculation of electromagnetic field radiated by power converters circuits using a numerical method based on the resolution of Maxwell equations by FDTD method in 3D and an analytical method using the Hertzian dipoles method. This last model, valid in case of thin wire theory and simple to implement, gives comparable results to those obtained by the first one and also the moment's method (FEKO). The resolution of the Maxwell equations by FDTD requires meshing of the volume with different materials (air, dielectric, conductor...). The choice of the step space in 3D is imposed by the stability criterion [10] required when using the FDTD. Considering the fast switches used in the power electronics circuits, choosing an appropriate step time impose a fine spatial discretization, which inevitably leads to computation time high and sometimes numerical noises. Using the analytical concept of Hertzian dipoles we can quantify by calculating the magnitude of the electromagnetic field emitted and keeps the spectral content of the signal without great inconvenience.

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